Approval: 22nd Senate Meeting

Course Name : Advanced Partial Differential Equations

Course Number : MA 603

Credit : 3-0-0-3

Prerequisites : MA 522 Partial Differential Equation

Students intended for : M.Sc./B.Tech/M.S./M.Tech/Ph.D.

Elective or core :Elective

Semester : Odd/Even

Preamble: This is a course intended to introduce the advanced concepts of partial differential equations. Through a detailed study of Greens function, students will learn the advanced analytical techniques available to solve nonlinear PDEs. Later they will be made to understand the role of Sobolev norms and compact embeddings to solve PDEs and find spectral decompositions.

Course Content:

Module 1: Green's Function: Green's functions, Green's functions and applications for Laplace, Poisson and Helmholtz equations Green's functions and applications for the heat equation Green's functions and applications for the wave equation. [4 hours]

Module 2: Non Linear First Order PDE: Complete Integrals and New Solutions from Envelopes, Local Solution and Application. Equations that convert into linear PDE; some exactly solvable cases; Burgers' equation; dimensional analysis and similarity; travelling waves; nonlinear diffusion and dispersion. Introduction of Hamilton Jacobi Equations, Calculus of Variations, Hamilton's O.D.E., Boundary Conditions, Local Solutions and Applications. [6 hours]

Module 3: Asymptotics, Singular perturbations, Turing Instability for Reaction Diffusion System, Laplace's Method, Homogenization, Power Series, Non Characteristic Surfaces, Real analytic Functions, Cauchy Kovalevskaya Theorem. [3hours]

Module 4: Sobolev Spaces: Introduction to Hilbert Spaces of Functions, Holder Spaces, Sobolev spaces; Definitions and Elementary Properties. Weak solution, Uniqueness and Properties of Weak derivatives. Definition & Properties of Sobolev Spaces, Inequalities, Compactness.Extensions,Sobolev. [12 hours]

Module 5: Second Order Elliptic Equations: Weak Solutions of Elliptic Equation, Existence of weak solutions, Regularity, Maximum Principles, Eigen values & Eigen Functions of symmetric elliptic operators. [8 hours]

Module 6: **Reaction Diffusion System**: Weak Solution of Diffusion Equation, Green's Function of Diffusion Equation, Formulation of Reaction Diffusion models and extensions to Include Chemotaxis terms; Application of Reaction Diffusion Systems to Population Dynamics, Pattern and Wave Phenomenon in the Life Sciences, Semi-arid vegetation and wound healing asPrototype Examples [9 hours]

Similarity content declaration with existing courses:

Sl. No.	Course Code	Similarity Content	Approximate % of Content
1	MA 522	Green's Function	Less than 5%

Justification for new course proposal if cumulative similarity content is > 30%:

Not Applicable

Text Book:

1. Evans, Lawrence C. *Partial Differential Equations*. Graduate Studies in Mathematics, vol. 19. Providence, RI: American Mathematical Society, 2010. ISBN: 9780821807729.
2. Brezis, H. And H. Brezis. 2011,' Functional Analysis, Sobolev Spaces and Partial Differential Equations' Newyork: Springer.

Reference Books:

1. Debnath, Lokenath. *Nonlinear partial differential equations for scientists and engineers*. Springer Science & Business Media, 2011.
2. DiBenedetto, Emmanuele. *Partial Differential Equations*. Boston, MA: Birkhäuser, 1995. ISBN: 9780817637088.
3. Garabedian, Paul. *Partial Differential Equations*. Providence, RI: AMS Chelsea, 1998. ISBN: 9780821813775.
4. E.Kreyszig,Advanced Engineering Mathematics, Wiley, 2011.