IIT Mandi
Proposal for a New Course

Course number : MA-527
Course Name : Field and Galois Theory
Credit Distribution : 3-1-0-4
Intended for : M.Sc./M.S./PhD/B.Tech
Prerequisite : MA-549 (Abstract Algebra)
Mutual Exclusion : (None)

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1. **Preamble:** This one semester course is designed to provide exposure to the concepts of fileds and to teach students how one can use group theory to solve field theory problems. The main idea of Galois theory is to associate a group, the Galois group, to a field extension. Since the Galois group of a finite dimensional extension is finite, we can utilize the numerical information about finite groups to help investigate such field extensions. It turns out that field theory is the right context for solving some of the famous classical problems that stumped mathematicians for centuries. As an application of field theory, we give proofs of the famous impossibilities of certain ruler and compass constructions, and we determine why roots of polynomials of degree 5 or greater need not be given by formulas involving field operations and extraction of roots.

2. **Course Modules with quantitative lecture hours:**

   Module 1: Fields, Characteristics and prime subfields, Field extensions, Automorphisms, Normal extensions and Splitting fields, Separable and Inseparable extensions, Algebraic closures **(12 Hours)**

   Module 2: Galois groups, The fundamental theorem of Galois theory, Finite fields, Cyclotomic extensions, Composite extensions, Norm and Traces, Cyclic extensions, Hilbert theorem 90 and Group cohomology, Kummer extensions **(16 Hours)**

   Module 3: Discriminants, Polynomials of degree 3 and 4, Ruler and Compass constructions, Solvability by radicals, Polynomials with Galois group Sn, Transcendental extensions, Solution of a cubic by Cardan's method, Solution of biquadratic by Ferrari's method **(14 Hours)**

3. **Text books:**

   1. P. Morandi, Field and Galois Theory, Springer-Verlag, New York, 1996

2. D.S. Dummit and R. M. Foote, Abstract Algebra, 2nd Edition, John Wiley, 2002.

4. References:

1. M. Artin, Algebra, Prentice Hall of India, 1994.
2. N. Jacobson, Basic Algebra I, 2nd Edition, Hindustan Publishing Co., 1984, W.H. Freeman, 1985.
3. S. Lang, Algebra, 3rd Edition, Springer (India), 2004.
4. J.S. Milne, Online notes : https://www.jmilne.org/math/CourseNotes/FT.pdf

5. Similarity with the existing courses:

(Similarity content is declared as per the number of lecture hours on similar topics)

S. No.	Course Title	Course Code	Similarity Content	Approx. % of Content
1.	Definition of a field, finite and algebraic extension	MA549	3 Hours	~7%