

General Relativity: From Christoffel Symbols to the Riemann Tensor

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The goal of this talk is to derive a means of calculating the intrinsic curvature of a manifold: the Riemann Curvature Tensor. We begin with an introduction to covariant differentiation, showing that since the basis vectors of a coordinate system change, we must utilize a correction term to obtain a derivative-like formulation for covariant vectors, known as the covariant derivative: $\nabla_r V_m = \partial_r V_m - \Gamma_{rm}^t V_t$. Much like derivatives are the generators of infinitesimal translations, covariant derivatives generate infinitesimal parallel transport of vectors. We subsequently derive the equation for the Christoffel symbols, assuming a torsion-free physical system. By exploiting the symmetry of the covariant indices, we find that $\Gamma_{mn}^t = \frac{1}{2}g^{rt}[\partial_n g_{rm} + \partial_m g_{rn} - \partial_r g_{mn}]$. We also discuss the covariant derivative of a mixed tensor, $\nabla_\mu T_\nu^\lambda = \partial_\mu T_\nu^\lambda + \Gamma_{\alpha\mu}^\lambda T_\nu^\alpha - \Gamma_{\mu\nu}^\sigma T_\sigma^\lambda$. By combining $\nabla_\mu T_\nu^\lambda$ and $\nabla_r T_m$, we obtain the Riemann Curvature Tensor, $R_{srn}^t = \partial_r \Gamma_{sn}^t - \partial_s \Gamma_{rn}^t + \Gamma_{sn}^p \Gamma_{pr}^t - \Gamma_{rn}^p \Gamma_{ps}^t$. We summarize this result in the context of General Relativity, and demonstrate the four steps to deriving the metric for a given spacetime: calculate the Christoffel symbols, substitute it into the Riemann curvature tensor, contract it into the Ricci tensor, and finally substitute it into Einstein's field equation.

¹Lecture at IIT Mandi on August 17, 2024

Curriculum Vitae

Suborno Isaac Bari

NYU Math & Physics BS Candidate
Youngest NYU Student in 193 years
Youngest Perfect AP Calculus BC Scorer

Birth: April 9, 2012, New York, NY

Citizenship: American

Address: 62 Edmund Street, Lynbrook NY 11563

Education: NYU (BS, 2024-2026)
Malverne High School (HS Diploma, 2022-2024)

Awards & Recognition:

[CNN](#), “This 12-year-old memorized the periodic table at age 2. He’s heading to NYU.”
[FOX](#), “Suborno Bari is the youngest person ever accepted to NYU”
[The Washington Post](#), “He graduated from HS at age 12. Now he’s heading to NYU.”
[Recognition from President Obama](#) (2016)
[Global Child Prodigy Award](#) from Nobel Laureate Kailash Satyarthi (2020)
[CBS](#) | [ABC](#) | [NBC](#) | [FOX & Friends](#) | [The New York Post](#)

Accomplishments:

AMC 12: 93 (Youngest AIME Qualifier in US Math Olympiad History)
SAT: 1500, ACT: 34, AP Calculus BC: 5, AP Physics 1: 4
Regents: Chemistry: (91), Geometry (87), Physics (97) and English (95)

Teaching:

[Jawaharlal Nehru University](#), [Bodoland University](#) (2024)
[United Group of Institutes](#), [Calcutta University](#), [Marathwada Krishi University](#) (2024)
[Christ University](#), [Jijau Dnyantirth](#), [Ruia College](#) (2019)
[Jain University](#), [IIT Jodhpur](#) (2023)

Books:

[The Love](#) (2019) | Published by AuthorHouse, United States
[Manish](#) (2023) | Published by KidsChaupal, India

OBJECTIVE: I'm a 12 Y/O B.S. student in math and physics at New York University. I hold three world records: 1) Youngest AIME qualifier in US Math Olympiad History, 2) Youngest ACT Math perfect scorer (36/36) & 3) Youngest AP Calculus BC perfect scorer (5/5) and youngest NYU student since it was founded nearly 200 years ago.

EDUCATION:

1. New York University | B.S., in Math and Physics (Expected graduation 2026)
2. Malverne High School | High School Diploma | Graduation May 2024

SCHOLARSHIPS:

CMT Scholarship 2023 from NYU Courant Institute

CAS Scholarship 2024 from NYU

NMSC Selection Index Score 219 (48-228). I met requirements for the National Merit Scholarship Program

ACCOMPLISHMENTS

1. Math Competition: 2023 AMC 12A: 93 (Youngest AIME Qualifier in Math Olympiad History)
2. Standardized Tests: PSAT 1470 | SAT score - 1500 | ACT: 34/36 (Perfect 36/36 ACT Math score)
3. Advanced Placement: Calculus BC score - 5, AP Physics I score - 4
2. Regents: Chemistry (91), Geometry (87), Physics (97) and English (95) [I took them between 9 and 10 Y/O]

TEACHING EXPERIENCE**Physics Lecture at Jawaharlal Nehru University and Bodoland University****2/2024**

I derived the equations of motion for a light sail, and demonstrated how the conservation of energy and momentum & Hamilton's optomechanical analogy lead directly to the equation for relativistic reflection.

Physics Lecture at Calcutta University**1/2024**

I delivered math and physics lectures at the [United Group of Institutes](#) in Uttar Pradesh, [Calcutta University](#) in Kolkata, [Marathwada Krishi University](#) (MWU) in Maharashtra, [Christ University](#) in Bangalore, and [Jijau Dnyantirth](#) in Parbhani.

Physics Teaching, Ruia College, University of Mumbai**1/2019**

[Taught](#) physics as a 7 Y/O at Ruia College of Mumbai University. Received recognition from Principal, Dr. Anushree Lokur.

Physics Teaching, Jain University**3/2023**

Delivered [a lecture on light](#) to hundreds of faculty and students. Received [recognition](#) from the university.

Physics Teaching, Indian Institute of Technology (IIT Jodhpur)**4/2023**

Created a simulation and website on black holes. Prepared course materials such as abstracts and handouts. [Taught](#) an interactive lecture on blackholes. Received [recognition](#) from Dr. Santanu Chaudhury, Director of IIT Jodhpur.

BOOKS

[The Love](#) | Published by AuthorHouse, United States

2019

I launched a campaign against hate entitled "The Love". This book received [The Vice Chancellor of Pune Award](#) from India

[Manish](#) | Published by KidsChaupal, India

2023

On January 4th, 2020, Principal Lokur of Ramnarain Ruia College invited a 7 year old boy to lecture the students at her college on math, physics and computer science. This book won an [award](#) from The Da Vinci Institute, South Africa.

RECOGNITION:

[CBS](#) | [CNN](#) | [FOX](#) | [ABC](#) | [NBC](#) | [FOX & Friends](#) | [NYPost](#) | [The Washington Post](#). Recognition from [President Obama](#) (2016), [Global Child Prodigy Award](#) from Nobel Laureate Kailash Satyarthi (2020) and Laureate Award from The [Da Vinci Institute](#), South Africa (2021) and from [Indian Schools](#) in Oman (2023). NBC Little Big Shots with [Melissa McCarthy](#).

RESEARCH

Collaborated with [Dr. Daniel Kabat](#) and [Dr. John Chiarelli](#) to write a paper on the *Parity of Perimeter Function for Regular Polygons*.